An Example of Midterm Exam - MGM1201

By Umi Mahnuna Hanung (*uploaded to website on 10th March, 2017*)

Name:________________________________

Student ID:____________________________

Section:_______________________________

Instructor:_____________________________

- This is a closed-book, closed-notes exam. No calculators or other electronic aids will be permitted.
- In order to receive full credit, please show all of your work and justify your answer. You do not need to simplify your answers unless specifically instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.
- Please sign the following:

> "On my honor, I have neither given nor received any aid on this examination. I have furthermore abided by all other aspects of the honor code with respect to this examination."

Signature: ___________________________

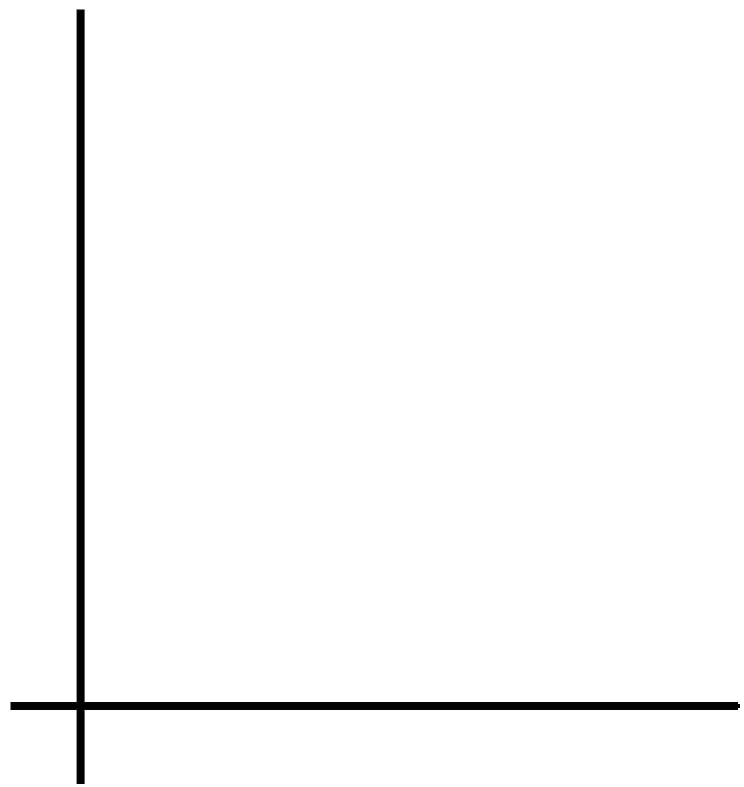
Score

Problem 1		Problem 13	
Problem 2		Problem 14	
Problem 3		Problem 15	
Problem 4		Problem 16	
Problem 5		Problem 17	
Problem 6		Problem 18	
Problem 7		Problem 19	
Problem 8		Problem 20	
Problem 9		Problem 21	
Problem 10		Problem 22	
Problem 11		Problem 23	
Problem 12		Total score	

Problem 1

(15 points) Let $f(x) = 6 - x^2$.

(a) On the axes below, sketch a graph of f over the domain $[0, 2]$, and then draw the approximating rectangles that are used to estimate the area under the curve (and above the y-axis) between $x = 0$ and $x = 2$ according to the *Midpoint Rule*; use $n = 4$ rectangles.



Problem 1 (Continued)

(b) Write an expression involving only numbers that represents the area estimate using these rectangles. (You do *not* have to expand or simplify the expression!)

(c) Find the exact area of the same region by evaluating the limit of a Riemann sum that uses the *Right Endpoint Rule*. (That is, do not use the Fundamental Theorem of Calculus.) Show all reasoning.

Problem 2

(8 points)

Suppose a study indicates that t years from now, the level of carbon dioxide in the air of a certain city will be

$$L(t) = (t + 1)^2$$

parts per million (ppm).

a. What is the average level of carbon dioxide in the first 3 years?

b. At what time (or times) does the average level of carbon dioxide actually occur? Answer to the nearest month.

Problem 3

(10 points)

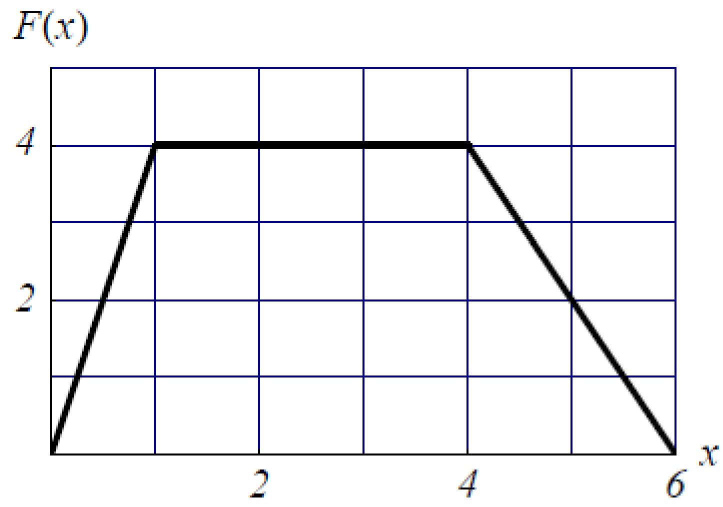
(a) Verify the following indefinite integral expression by differentiating.

$$\int \sqrt{1-x^2}\, dx = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x + C$$

(b) Use the *above formula* to compute the area of a *semicircle* of radius 1, centered at the origin. Show all steps in your calculation.

Problem 4

(5 points) Find a function $g(x)$ such that the graph of $F(x) = \int_0^x g(t)\,dt$ is the graph below.

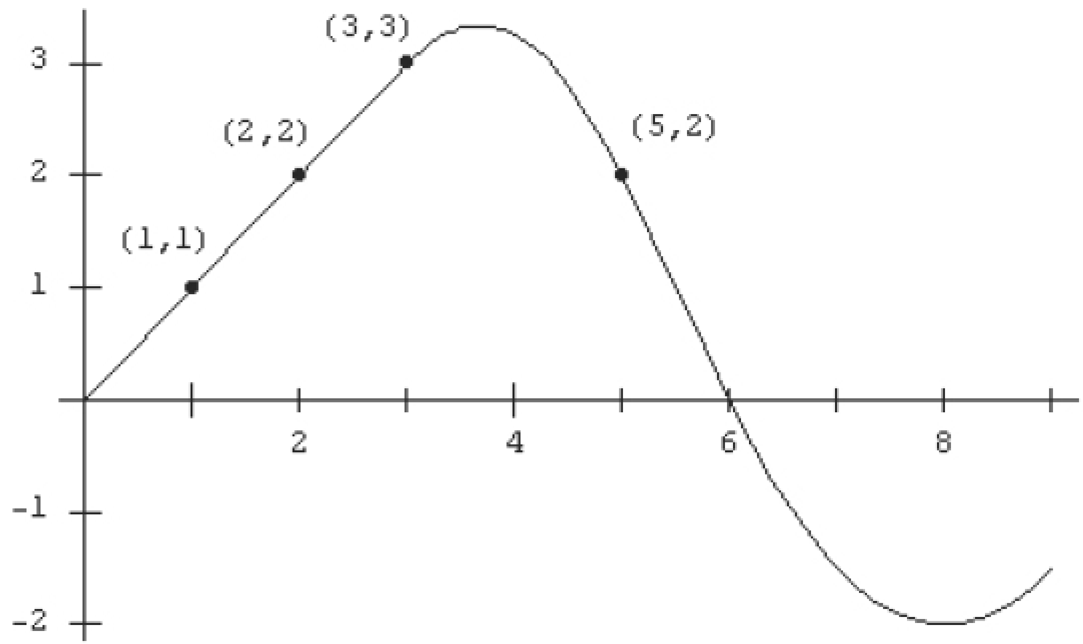


(Specify such a $g(x)$ on the domain [0,6].)

Problem 5

. (15 points) Let $s(t)$ be the position, in meters, at time t seconds of a particle moving along a coordinate axis, and *suppose the position at time 0 is 1 m* (i.e., $s(0) = 1$ m).

As usual, we write $v(t)$ for the velocity function (in meters per second). Suppose the graph of the *velocity function* $v(t)$ is shown below:



Use the information above to answer the following questions. *Give reasons for your answers.*

(a) What is the particle's velocity at time $t = 5$?

Problem 5 (Continued)

(b) Is the acceleration of the particle at time $t = 5$ positive or negative?

(c) What is the particle's position at time $t = 3$?

(d) At what time during the first 9 seconds does s have its largest value?

(e) Approximately when is the acceleration zero?

(f) When is the particle moving toward the origin? Away from the origin?

Problem 5 (Continued)

(g) On which side (positive or negative) of the origin does the particle lie at time $t = 9$?

(h) Given that $\int_0^6 v(t)\,dt = 11.5$ and $\int_6^9 v(t)\,dt = -4.5$, find the *total distance traveled* by the particle in the first 9 seconds.

Problem 6

(23 points) Show all reasoning when solving each of the problems below.

(a) $$\int \frac{dx}{\sqrt{-x^2 + 3x - 2}}$$

(b) $$\int_{1/\sqrt{3}}^{1} \frac{dx}{x\sqrt{4x^2 - 1}}$$

(c) $$\int_{0}^{\ln 2\sqrt{3}} \frac{e^x\, dx}{e^{2x} + 4}$$

Problem 6 (Continued)

(23 points) Show all reasoning when solving each of the problems below.

(d) $\displaystyle\int \frac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx$

(e) $\displaystyle\int \frac{dx}{x - x(\ln x)^2},\ |x| < e$

Problem 7

(10 points)

(a) Show that $0 \leq \int_0^1 x^4 \sin x \, dx \leq \frac{1}{5}$.

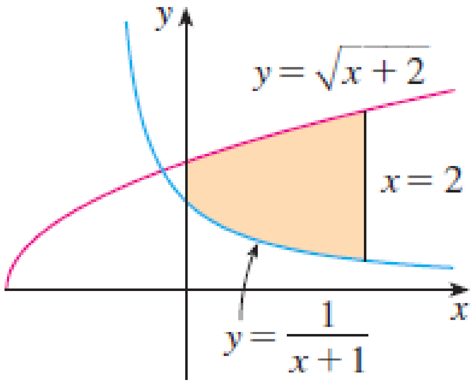
(b) Suppose $f'(x) = \frac{\cos x}{x}$, and let $f\left(\frac{\pi}{2}\right) = a$ and $f\left(\frac{3\pi}{2}\right) = b$ for constants a and b. Find the value of the integral

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} f(x) \, dx$$

Your answer will involve a and b.

Problem 8

(4 points) Find the area of the shaded region.



Problem 9

(4 points)

For what values of m do the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region? Find the area of the region.

Problem 10

(6 points) Mark each statement below as *true* or *false* by circling TRUE or FALSE. *No justification is necessary.*

(a) If g is an even function that is continuous at all values, then $\int_{-a}^{a} xg(x)\,dx = 0$ foı any value of a. TRUE FALSE

(b) If $f(x)$ is a differentiable function, then $\frac{d}{dx}\int_{a}^{x} f(t)dt = f(x)$ TRUE FALSE

(c) Let $F(x) = \int_{3}^{2x^2} \frac{\sin(t)}{t^3+1}\,dt$ be defined for $x > -1$. Then TRUE FALSE

$F'(x) = \frac{4x\sin(2x^2)}{8x^3+1} - \frac{12\sin(3)}{28}$